KOMPLEKS SONLARNING GEOMETRIK VA TRIGONOMETRIK SHAKLI, MUAVR FORMULASI.

Toshkent Moliya instituti akademik litseyi o'qituvchisi.
Maxmudaliev Faxriddin Furxoniddin o'g'li.
Sadriddinov Dilshod.
Jahon iqtisodiyoti va diplomatiyasi universiteti akademik litseyi o'qituvchisi.
Abduraximov Ilxomjon.

Annotatsiya. Ushbu maqolada Kompleks sonlar, uning ma'nolari va tadbiqlari haqida fikr yuritilgan.

Аннотация. В этой статье обсуждается Комплексные числа, его значения и приложения.

Annotation. This article discusses Complex Numbers, its meanings and applications.

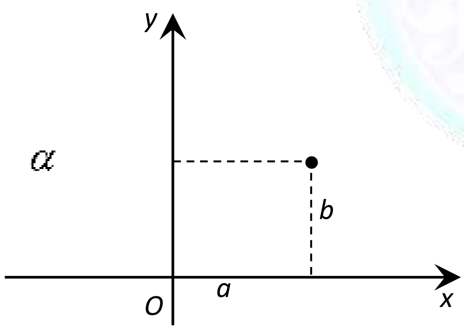
Kalit so'zlar: kompleks sonlar, vector, koordinata, Muavr formulasi, argument.

Ключевые слова: комплексные числа, вектор, координата, формула Муавра, аргумент.

Keywords: complex numbers, vector, coordinate, Moivre formula, argument.

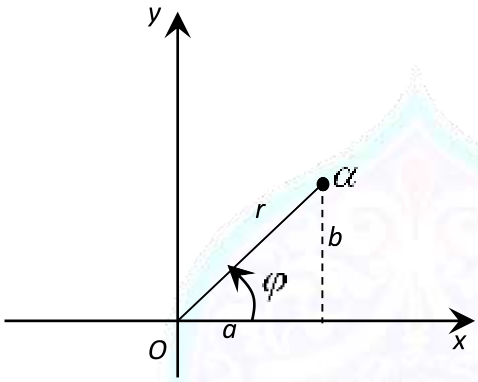
Agar haqiqiy sonlar to'plamini to'g'ri chiziq sifatida geometrik talqinini (ma'nosini) qarasak, u holda $\square^2$ ni tekislik deb qarashimiz mumkin.

$\square^2$ to'plam elementi bilan G kompleks sonlar maydonidagi kompleks sonlar o'rtasida o'zaro bir qiymatli moslik mavjuddir. Bu moslik (biyeksiya) $\square$ kompleks sonlar maydonining geometrik talqini (ma'nosi), tekislik esa kompleks tekislik deyiladi. Kompleks tekislikning abssissasi o'qi nuqtalariga kompleks sonning haqiqiy qismi sonlari, sof mavhum sonlarga esa ordinata o'qining nuqtalari mos keladi. Shuning uchun kompleks tekislikning abssissa o'qiga haqiqiy o'q, ordinata o'qiga esa mavhum o'q deyiladi va demak $\alpha = a + bi$ kompleks sonning kompleks tekislikdagi o'rni quyidagi shaklda tasvirlanadi:



1-shakl

Ko'p hollarda α nuqtani vektor sifatida qaralib, bu vektorni o'zi qutb koordinatalaridagi ifodasi ko'riladi.



2-shakl

Bu yerda r vektorning uzunligi (qutb radius) va φ burchak qutb burchak vektorning Ox o'qi bilan (soat strelkasiga qarama-qarshi yo'nalish musbat burchak, agar soat strelkasi bo'yicha yo'nalgan bo'lsa, manfiy burchak) tashkil etgan burchak deyiladi. Endi biz $\alpha = a + bi$ kompleks son bilan uning qutb koordinatalari orasidagi bog'lanishni 2 shakldagi to'g'ri burchakli uchburchakdan topamiz. Pifagor teoremasiga asosan

$$r^2 = a^2 + b^2 \quad \textbf{(1)}$$

$$r = \sqrt{a^2 + b^2} \quad \textbf{(2)}$$

tenglik orqali kompleks sonning (vektorning) uzunligi topiladi. Kosinus, sinus va tangenslarning ta'rifidagi φ burchak (π yoki 2π aniqligida) topiladi:

$$\cos\varphi = \frac{a}{r}, \ \sin\varphi = \frac{b}{r} \quad \textbf{(3)}$$

yoki

$$tg\varphi = \frac{b}{a} \quad \textbf{(4)}$$

tengliklardan topiladi.

Kompleks sonning uzunligiga (vektorning uzunligi yoki qutb radiusi) uning moduli deyiladi va $|\alpha|$ orqali belgilanadi. Kompleks sonning (vektorning) argumenti deb, uning Ox o'qi bilan tashkil etgan burchagiga aytiladi va $\arg\alpha$ orqali belgilanadi. Kompleks sonning burchagi φ bo'lsa, har qanday $k \in \mathbb{Z}$ uchun $\varphi + 2\pi k$ ham argumenti bo'ladi. Bu argumentga kompleks sonning katta argumenti deyiladi va Arg shaklda yoziladi.

Agar (3) tengliklarda a va b larni topsak:

$$a = r\cos\varphi, \ b = r\sin\varphi$$

va bularni kompleks sonning algebraik shakliga olib borib qo'ysak,

$$\alpha = r(\cos\varphi + i\sin\varphi) \qquad \textbf{(5)}$$

tenglikni hosil qilamiz. Bu tenglikka α kompleks sonning trigonometrik shakli deyiladi va bu tenglik yagona ravishda aniqlangandir (ko'rsating!).

Tabiiyki, kompleks sonning qo'shmasining trigonometrik shakli:

$$\bar{\alpha} = r(\cos\varphi - i\sin\varphi)$$

bo'ladi.

Teorema 15.1. Ikkita kompleks sonning ko'paytmasi moduli ko'paytuvchilar modullarining ko'paytmasi, argumenti ko'paytuvchilar argumentlarining yig'indisiga teng, ya'ni

$$|\alpha\cdot\beta| = |\alpha|\cdot|\beta| \qquad \textbf{(6)}$$

$$\arg\alpha\cdot\beta = \arg\alpha + \arg\beta. \qquad \textbf{(7)}$$

Isbot. Bizga $\alpha = r(\cos\varphi + i\sin\varphi)$ va $\beta = \rho(\cos\varphi + i\sin\varphi)$ kompleks sonlarining trigonometrik shakllari bo'lsin. U holda

$$\alpha\cdot\beta = r(\cos\varphi + i\sin\varphi)\rho(\cos\psi + i\sin\psi) =$$

$$= r\cdot\rho\big(\cos\varphi\cdot\cos\psi - \sin\varphi\cdot\sin\psi + i(\cos\varphi\cdot\sin\psi + \sin\varphi\cdot\cos\psi)\big) =$$

$$= r\cdot\rho\big(\cos(\varphi+\psi) + i\sin(\varphi+\psi)\big)$$

hosil bo'lib,

$$|\alpha\cdot\beta| = r\cdot\rho = |\alpha|\cdot|\beta|$$

va

$$\arg\alpha\cdot\beta = \varphi + \psi = \arg\alpha + \arg\beta$$

bo'ladi.

Bu teoremadan bevosita quyidagi natijani hosil qilamiz.

Natija 15.2. Bir nechta $\alpha_1, \alpha_2, ..., \alpha_n$ sonlarning ko'paytmasining moduli

$$|\alpha_1\cdot\alpha_2\cdot...\cdot\alpha_n| = |\alpha_1|\cdot|\alpha_2|\cdot...\cdot|\alpha_n| \qquad \textbf{(8)}$$

va argumenti

$$\arg(\alpha_1\cdot\alpha_2\cdot...\cdot\alpha_n) = \arg\alpha_1 + \arg\alpha_2 + ... + \arg\alpha_n \qquad \textbf{(9)}$$

bo'ladi.

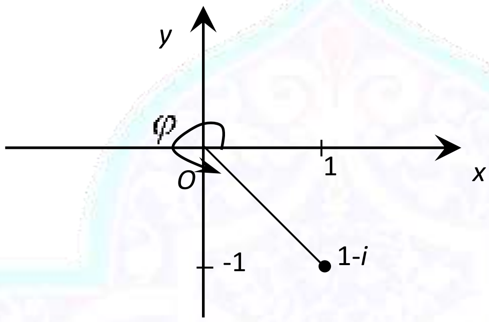
Misol. $\alpha = 1 - i$ ni geometrik o'rni va trigonometrik shaklini ko'rsating.

$$a = 1,\ b = -1$$

$$r = \sqrt{1+1} = \sqrt{2},\ tg\varphi = \frac{-1}{1} = -1 \qquad \varphi = arctg(-1) = \frac{7\pi}{4}$$

va demak

$$\alpha = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right).$$



Misol. $\alpha = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ va $\beta = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ ko'paytmasi

$$\alpha\cdot\beta = 2\sqrt{2}\left(\cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{4}+\frac{\pi}{3}\right)\right) =$$

$$= 2\sqrt{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$$

bo'ladi.

Natija 15.3. (Muavr formulasi) Har qanday $n \in$ □ son $\varphi \in$ □ uchun

$$\alpha^n = r^n\left(\cos n\varphi + i\sin n\varphi\right) \qquad \textbf{(9)}$$

bo'ladi,

$$\left|\alpha^n\right| = \left|\alpha\right|^n,\ \arg\alpha^n = n\arg\varphi,$$

ya'ni kompleks soni darajaga ko'tarishda modul shu darajaga ko'tariladi, argument esa darajaga ko'paytiriladi.

Isbot. (7) va (8) formulalarda

$$r_1 = r_2 = \ldots = r_n = r \text{ va } \varphi_1 = \varphi_2 = \ldots = \varphi_n = \varphi$$

deb olsak, (9) formulaning natural sonlar uchun o'rinli ekanligi kelib chiqadi.

$$\alpha^{-1} = \frac{1}{\alpha} = \frac{1}{r\left(\cos\varphi + i\sin\varphi\right)}\cdot\frac{\cos\varphi - i\sin\varphi}{\cos\varphi - i\sin\varphi} =$$

$$= \frac{1}{r}\frac{\cos\varphi - i\sin\varphi}{\cos^2\varphi + i\sin^2\varphi} = r^{-1}\left(\cos\varphi - i\sin\varphi\right) = \qquad \textbf{(10)}$$

$$= r^{-1}\left(\cos(-\varphi) + i\sin(-\varphi)\right)$$

tenglik (9) formulaning $n=-1$ da o'rinli ekanligi ko'rsatadi. Endi ixtiyoriy n manfiy butun son uchun $n=-m$, $m\in\mathbb{N}$ deb olib,

$$\left(r\left(\cos\varphi+i\sin\varphi\right)\right)^{n}=\left(r\left(\cos\varphi+i\sin\varphi\right)\right)^{-m}=$$
$$=\left(\left(r\left(\cos\varphi+i\sin\varphi\right)\right)^{-1}\right)^{m}=$$
$$=\left(r^{-1}\left(\cos\left(-\varphi\right)+i\sin\left(-\varphi\right)\right)\right)^{m}=$$
$$=\left(r^{-1}\right)^{m}\left(\cos\left(-m\varphi\right)+i\sin\left(-m\varphi\right)\right)=$$
$$=r^{-m}\left(\cos n\varphi+i\sin n\varphi\right)=$$
$$=r^{n}\left(\cos n\varphi+i\sin n\varphi\right),$$

ya'ni (9) tenglik n manfiy butun son uchun ham o'rinli.

(9) tenglikga Muavr formulasi deyiladi.

$$\left(1-i\right)^{10}=\left(\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)\right)^{10}=$$
$$=\left(\sqrt{2}\right)^{10}\left(\cos 10\cdot\frac{7\pi}{4}+i\sin 10\cdot\frac{7\pi}{4}\right)=$$

Misol. $$=2^{5}\left(\cos\frac{35\pi}{2}+i\sin\frac{35\pi}{2}\right)=$$
$$=32\left(\cos\left(16\pi+\frac{3\pi}{2}\right)+i\sin\left(16\pi+\frac{3\pi}{2}\right)\right)=$$
$$=32\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)=32\cdot\left(-i\right)=-32i$$

Natija 15.4. Ikkita kompleks sonning nisbatining modullari modullar nisbatiga, argumenti argumentlar ayirmasiga teng.

Isbot. $\alpha=r\left(\cos\varphi+i\sin\varphi\right)$ va $\beta=\rho\left(\cos\psi+i\sin\psi\right)$ kompleks sonlarning trigonometrik shakli berilgan bo'lsin. U holda

$$\frac{\alpha}{\beta}=\alpha\cdot\beta^{-1}=\left(r\left(\cos\varphi+i\sin\varphi\right)\right)\left(\rho\left(\cos\psi+i\sin\psi\right)\right)=$$
$$=\left(r\left(\cos\varphi+i\sin\varphi\right)\right)\left(\rho^{-1}\left(\cos\left(-\psi\right)+i\sin\left(-\psi\right)\right)\right)=$$
$$=r\cdot\rho^{-1}\left(\cos\left(\varphi-\psi\right)+i\sin\left(\varphi-\psi\right)\right)$$

bo’lib, bundan

$$\left|\frac{\alpha}{\beta}\right| = r\cdot\rho^{-1} = \frac{r}{\rho} = \frac{|\alpha|}{|\beta|} \quad \text{va} \quad \arg\frac{\alpha}{\beta} = \varphi-\psi = \arg\varphi - \arg\psi$$

bo’ladi. **Misol**.

$$\frac{1-i}{1+\sqrt{3}i} = \frac{\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)}{2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)} =$$

$$= \frac{\sqrt{2}}{2}\left(\cos\left(\frac{7\pi}{4}-\frac{\pi}{3}\right)+i\sin\left(\frac{7\pi}{4}-\frac{\pi}{3}\right)\right) =$$

$$= \frac{\sqrt{2}}{2}\left(\cos\frac{17\pi}{12}+i\sin\frac{17\pi}{12}\right)$$

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