EYLER ALMASHTIRISHLARINING RATSIONAL FUNKSIYALARNI INTEGRALLASHDAGI TADBIQLARI

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ANNOTASIYA

Ushbu ishda Eyler almastirishlari va uning uchta holatiga oid misollar ishlab tushintirilgan. Eyler almashtirishlarining kundalik hayotdagi ahamiyati yoritilgan.

Kalit so'zlar: Eyler, funksiya, integral, ratsional.

Ushbu tezisda Eyler almashtirishlarning uchta holati o'rganilgan. $\int R(x, \sqrt{ax^2 + bx + c}\,)dx$ ko'rinishdagi integrallar Eyler almashtirishlaring uchta holatiga qo'yish orqali ratsional funksiyalardan olinadigan integrallarga keltiriladi:

$$1.1 \quad \begin{cases} a > 0 \\ D < 0 \end{cases} \quad \sqrt{ax^2 + bx + c} = t \pm \sqrt{a}x$$

$$2.1 \quad \begin{cases} a < 0 \\ c > 0 \\ D < 0 \end{cases} \quad \sqrt{ax^2 + bx + c} = tx \pm \sqrt{c}$$

$$3.1 \quad D > 0 \quad \sqrt{ax^2 + bx + c} = t(x-x_1)$$

$$\sqrt{ax^2 + bx + c} = t(x-x_2)$$

$$ax^2 + bx + c = a(x-x_1)(x-x_2)$$

1.1 Agar $a > 0$ bo'lsa,

$\sqrt{ax^2 + bx + c} = \pm\sqrt{a}x + t$ almashtirish qilamiz. U holda $ax^2 + bx + c = ax^2 + 2\sqrt{a}tx + t^2$ bo'ladi. Bundan x ni t ning ratsional funksiyasi sifatida aniqlaymiz.

$$x = \frac{t^2 - c}{b - 2\sqrt{a}t}$$

Bu yerda dx ham t ning ratsional funksiyasidan iborat bo'ladi. Shunday qilib $\sqrt{ax^2 + bx + c} = \sqrt{a}\,\frac{t^2 - c}{b - 2\sqrt{a}t} + t$ bo'lib u t ni ratsional funksiyasi bo'ladi.

$\sqrt{ax^2+\text{bx}+\text{c}}$ = t $\pm\sqrt{a}$x almashtirish orqali integral ostidagi funksiya ratsionallashtiriladi.(Eylerning birinchi holati)

1.2-misol $\int\frac{dx}{x\sqrt{x^2+x+1}}$ integralni hisoblang.

Yechish: Funksiya ratsional funksiya bo'lganligi uchun Eyler almashtirishlariga binoan integralni hisoblaymiz. a>0 bo'lganligi uchun funksiyani Eylerning birinchi almashtirishiga binoan yozamiz

$$\sqrt{x^2+x+1}=\text{t+x}$$

ifodaning ikkala tarafini kvadratga oshiramiz

$$x^2+x+1=t^2+2tx+x^2$$

$$1-t^2=(2t-1)x$$

$$x=\frac{1-t^2}{2t-1} \qquad (1)$$

hosil bo'lgan (1) ifodadan birinchi tartibli hosila olamiz

$$dx=\frac{-2t(2t-1)-2(1-t^2)}{(2t-1)^2}dt$$

$$dx=\frac{-4t^2+4t-2+2t^2}{(2t-1)^2}dt$$

$$dx=\frac{-2t^2+4t-2}{(2t-1)^2}dt \qquad (2)$$

(1)- va (2)- ifodalarni mos ravishda funksiyaga qo'yamiz

$$\int\frac{1}{\frac{1-t^2}{2t-1}*\left(1+\frac{1-t^2}{2t-1}\right)}*\frac{-2t^2+4t-2}{(2t-1)^2}\,dt=\int\frac{(2t-1)^2}{1-t^2*(2t^2-t+1-t^2)}*\frac{-2t^2+4t-2}{(2t-1)^2}\,dt=$$

$$\int\frac{-2(t^2-2t+1)}{(1-t)(1+t)*(t^2-t+1)}dt=\int\frac{2(t-1)}{(1+t)*(t^2-t+1)}dt=$$

Hosil bo'lgan ifodani sodda kasrlarga yoyamiz va A, B koeffitsientlarni topamiz.

$$\frac{A}{t+1}+\frac{B}{t^2-t+1}=\frac{2(1-t)}{(t+1)*(t^2-t+1)}$$

$$\text{A}(t^2-t+1)+\text{B}(t+1)=2t-2$$

A va B koeffitsientlarni t parametrga tanlash usuli bilan qiymat berish orqali aniqlaymiz

<u>t=-1</u> 3A=-4 <u>A</u>= $\frac{-4}{3}$

<u>t=0</u> $\frac{-4}{3}$ + B = -2 <u>B</u> = $\frac{-2}{3}$

Koeffitsientlarni mos ravishda ifodaga qo'yib integral olamiz.

$$\int\frac{-4}{3(t+1)}-\frac{2}{3(t^2-t+1)}\,dt=\frac{-4}{3}\ln|t+1|-\frac{2}{3}\int\frac{1}{t^2-t+1}\,dt=\frac{-4}{3}\ln|t+1|-$$

$$\frac{2}{3}\int\frac{1}{t^2-t+\frac{1}{4}+\frac{3}{4}}\,dt=\frac{-4}{3}\ln|t+1|-\frac{2}{3}\int\frac{1}{\left(t-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,dt=\frac{-4}{3}\ln|t+1|-\frac{2}{3}*\frac{2}{\sqrt{3}}arctg\frac{\left(t-\frac{1}{2}\right)*2}{\sqrt{3}}$$

$$=\frac{-4}{3}\ln\left|\sqrt{x^2+x+1}-\text{x}\right|-\frac{4}{3\sqrt{3}}arctg\frac{2*(\sqrt{x^2+x+1}-\text{x})-1}{\sqrt{3}}$$

Natija:

$$\int \frac{dx}{x\sqrt{x^2+x+1}} = \frac{-4}{3}\ln\left|\sqrt{x^2+x+1} - x\right| - \frac{4}{3\sqrt{3}} arctg \frac{2*(\sqrt{x^2+x+1} - x)-1}{\sqrt{3}}$$

2.1 Eylerning ikkinchi almashtirishi. Agar $c>0$ bo'lsa, $\sqrt{ax^2+bx+c}=xt\pm\sqrt{c}$ almashtirish qilamiz. Oxirgi tenglikni har ikkala tomonini kvadratga ko'tarsak $ax^2+bx+c=x^2t^2\pm 2xt\sqrt{c}+c$ tenglik hosil bo'ladi. Bu ifodadan $\sqrt{c}$ oldida plyus ishorani olib x ni topamiz,

$$x=\frac{2\sqrt{c}t-b}{a-t^2}$$

dx va $\sqrt{ax^2+bx+c}$ larni t orqali ifodalab berilgan integralga , x dx va $\sqrt{ax^2+bx+c}$ ning t orqali qiymatlarini qo'ysak integral ratsionallashadi.

c>0 bo'lganda $\sqrt{ax^2+bx+c}$ = tx ± $\sqrt{c}$ almashtirish yordamida integral ostidagi funksiya ratsionallashtiriladi.(Eylerning ikkinchi holati)

2.2 $\int\frac{dx}{3-\sqrt{9-x^2}}$ integralni hisoblang.

Yechish. Berilgan integra $\int R\left(x,\sqrt{a^2-x^2}\right)dx$ ko'rinishdagi integraldir. Bunda $x=a\sin t$ ko'rinishda almashtirish bajaramiz.

$$\int\frac{dx}{3-\sqrt{9-x^2}}=\left|\begin{matrix}x=3\sin t\\ dx=3\cos tdt\end{matrix}\right|=\int\frac{3\cos tdt}{3-\sqrt{9-(3\sin t)^2}}=\int\frac{3\cos tdt}{3-\sqrt{9\left(1-\sin^2 t\right)}}=$$

$$\int\frac{\cos tdt}{1-\cos t}$$

Oxirgi integralda trigonometrik almashtirishlardan foydalanamiz.

$$\int\frac{\cos tdt}{1-\cos t}=\left|tg\frac{t}{2}=y,\ \cos t=\frac{1-tg^2\frac{t}{2}}{1+tg^2\frac{t}{2}}=\frac{1-y^2}{1+y^2},\ dt=\frac{2dy}{1+y^2}\right|=\int\frac{\frac{1-y^2}{1+y^2}}{1-\frac{1-y^2}{1+y^2}}\cdot\frac{2dy}{1+y^2}=$$

$$=\int\frac{2dy}{2y^2\left(1+y^2\right)}=\int\frac{dy}{y^2\left(1+y^2\right)}=\int\left(\frac{1}{y^2}-\frac{1}{1+y^2}\right)dy=-\frac{1}{y}-arctgy+C=$$

$$=-\frac{1}{tg\frac{t}{2}}-arctg\left(tg\frac{t}{2}\right)+C=-ctg\frac{t}{2}-\frac{t}{2}+C=-ctg\left(\arcsin\frac{x}{3}\right)-\frac{1}{2}\cdot\arcsin\frac{x}{3}+C$$

Natija:

$$\int\frac{dx}{3-\sqrt{9-x^2}}=-ctg(\arcsin\frac{x}{y})-\frac{1}{2}\arcsin\frac{x}{3}+C.$$

3.1 α va β ax^2+bx+c kvadrat uchhadning haqiqiy ildizlari bo'lganda

$$\sqrt{ax^2+bx+c}=(x-\alpha)t$$

almashtirishni olamiz. U holda $ax^2+bx+c=a(x-\alpha)(x-\beta)$ bo'lgani uchun $\sqrt{a(x-\alpha)(x-\beta)}=(x-\alpha)t$ tenglik hosil bo'ladi. Bu tenglikni kvadratga ko'tarib x o'zgaruvchini topamiz va bundan $x=\frac{a\beta-\alpha t^2}{a-t^2}$ kelib chiqadi. dx va $\sqrt{ax^2+bx+c}$ larni t orqali ifodalab berilgan integralga , x dx va $\sqrt{ax^2+bx+c}$ ning t orqali qiymatlarini qo'ysak integral ratsionallashadi. (Eylerning uchinchi holati)

3.2 Integralni Eyler almashtirishlarning 3.1-holati orqali hisoblang.

3.2-misol. $\int\frac{dx}{\sqrt{x^2-3x+2}}$ integralni hisoblang.

Yechish: $x^2-3x+2=0$ - tenglamaning ildizlarini topamiz

(x-2)(x-1)=0 $x_1=2$, $x_2=1$

Tenglama ildizlaridan ixtiyori biri orqali funksiyani Eylerning 3-almashtirishi bo'yicha yozamiz.

$$\sqrt{x^2-3x+2}=t(x-2)$$

$$x^2-3x+2=t^2(x-2)^2$$

$$\frac{(x-2)(x-1)}{(x-2)^2}=t^2 \qquad t^2=\frac{x-1}{x-2}$$

$$t^2x-2t^2=x-1$$

$$t^2x-x=2t^2-1$$

$$x=\frac{2t^2-1}{t^2-1} \qquad (3)$$

hosil bo'lgan (3) ifodadan birinchi tartibli hosila olamiz.

$$dx=\frac{4t(t^2-1)-2t(2t^2-1)}{(t^2-1)^2}$$

$$dx=\frac{4t^3-4t-4t^2+2t}{(t^2-1)^2}=\frac{-2t}{(t^2-1)^2} \qquad (4)$$

(3) va (4) ifodalarni mos ravishda funksiyaga qo'yib integralni hisoblaymiz.

$$\int\frac{1}{t\left(\frac{2t^2-1}{t^2-1}-2\right)}*\frac{-2t}{(t^2-1)^2}dt=\int\frac{t^2-1}{2t^2-1-2t^2+2}*\frac{-2t}{(t^2-1)^2}dt=\frac{-2t}{t^2-1}dt=$$

$$=-\int\left(\frac{1}{t-1}-\frac{1}{t+1}\right)dt=-(\ln|t-1|-\ln|t+1|)=\ln\left|\frac{t+1}{t-1}\right|+C==\ln\left|\frac{\sqrt{\frac{x-1}{x-2}}+1}{\sqrt{\frac{x-1}{x-2}}-1}\right|+C$$

Natija:

$$\int\frac{dx}{\sqrt{x^2-3x+2}}=\ln\left|\frac{\sqrt{\frac{x-1}{x-2}}+1}{\sqrt{\frac{x-1}{x-2}}-1}\right|+C$$

Xulosa:

Xulosa qilib aytish mumkinki, Eyler almashtirishlar, matematikadagi bir necha funksiyalarni boshqa funksiyalarga aylantirish imkonini beradi. Bu matematikaning turli sohalarida ahamiyatli bir qo'shimcha qobiliyatdir.Birinchi navbatda, fizika va injeneriyada, eyler almashtirishlar turli turli integral operatorlar va differensial operatorlarini yechishda keng qo'llaniladi. Misol uchun, elektronikaning elektronik davomati tizimlarida, elektr to'qimalarida va termodinamikadagi qonuniyliklarda foydalanish mumkin.Bir necha funksiyalarni boshqa funksiyalarga aylantirishning aniqlik talab qiladigan xolatlarida, eyler almashtirishlaridan foydalaniladi. Misol uchun, statistikada bu almashtirishlar statistik kuzatuvlarda yoki hisob-kitobdagi ma'lumotlar analizida keng qo'llaniladi. Bu esa, matematikni turli sohalarida muhim bir qo'shimcha qobiliyat sifatida qaraladi.

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