Qavariq chegaralanmagan soxada bitta qochuvchi ikkita quvlovchi differensial o'yini

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Γ:(z,y_0-x_0)=(z_0, y_0-x_0) O`rta perpendikularlar tenglamasi

$X(x_0, y_0)$: (z,y_0-x_0)≥ ($z_0, (y_0-x_0)$) O`rta perpendikular ajratgan yarim tekisliklardan pastki yarim tekislik.

M-qavariq to`plam

A=$X_1(x_{10}, y_0) \cap X_2(x_{20}, y_0) \cap M$

Bizga A to`plamda quyidagi shartlar asosida differensial o`yin berilgan bo`lsin.

$$\begin{cases} \dot{x_i} = u_i, \ |u_i| \le 1, x_i \notin A, x_{i(0)} = x_{i0} \\ \dot{y} = v, |v| \le 1, y \in A, y(0) = y_0 \end{cases} \quad i=1,2 \qquad (1)$$

Teorema: Agar A to`plam chegaralangan bo`lsa u xolda (1) *shartlar asosida berilgan differensial* o`yinni tugatish mumkun:

Isbot:

Bizga (1) *shartlar asosida berilgan differensial o`yin*da $\Gamma_1$ va $\Gamma_2$ O`rta perpendikularlar o`zaro parallel,ustma-ust yoki kesishishi mumkun.

1-xol: $\Gamma_1$va Γ_2 O`rta perpendikularlar o`zaro parallel yoki ustma-ust bo`lsin. Bu xolda $A_i$=$X_i(x_{i0}, y_0) \cap M$ i=1,2 to`plamlardan kamida bittasi chegaralangan qavariq to`plam bo`ladi.Aniqlik $A_1 = X_1\,(x_{10}, y_0) \cap M$ *chegaralangan qavariq to`plam bo`lsin.*Bu xolda 2-quvlovchi qochuvchi bilan o`rta perpendikularga 1-marta parallel to`g`ri chiziqqa joylashib olguncha bor tezlikda xarakat qiladi.Bu vaqtda 1-quvlovchi $\pi$ strategiya bo`yicha xarakat qiladi.Aytaylik t_1 vaqtda shu xol ro`y bersin. . $t_1$ Vaqtdan keyin qochuvchi turgan nuqtani koordinata boshi,quvlovchilar va qochuvchi orqali o`tgan to`g`ri chiziqlarni kordinata o`qlari sifatida yangi koordinatalar sistemasi kiritib olamiz. $t_1$vaqtdan so`ng 1-xol uchun differensial o`yinni tugatish mumkin:

Isbot: 1)Quvlovchilar π strategiya bo`yicha xarakat qiladi

2) $|u_i| \leq 1$ $|v| \leq 1$ O`yin ishtirokchilarining tezliklari 1 dan ortib ketmaydi

3)O`yinni tugatish mumkunligi: $e_i = \frac{y_0 - x_{i0}}{|y_{0-}x_{io}|}$ , Qochuvchilar o`zaro perpendikular bo`lgani uchun :

$e_1 = (0, -1), e_2 = (-1, 0)$

$$(v, e_1) = (v_1, v_2) \cdot (0, -1) = -v_2$$

$$(v, e_2) = (v_1, v_2) \cdot (-1, 0) = -v_1$$

$\max(m - x_{i0}, e_i) = c_i$ bo`lsin

$$\sum_{i=1}^{2}(y(t) - x_i(t), e_i) = \sum_{i=1}^{2} y(t) - x_{i0}, e_i) - \sum_{0}^{t}(\sqrt{1 - v_1{}^2} + \sqrt{1 - v_2{}^2})ds$$

$$\leq c_1 + c_2 - \int_0^t (\sqrt{1 - v_1{}^2} + \sqrt{1 - v_2{}^2})ds$$

f$((v_1, v_2) = \sqrt{1 - v_1{}^2} + \sqrt{1 - v_2{}^2} \leq |v_2| + \sqrt{1 - v_2{}^2}$=f$(v_2)$

$|v_2| = z$,   f(z)=z+ $\sqrt{1 - z^2}$ $f(\dot{z}) = 1 - \frac{z}{\sqrt{1 - z^2}} \geq 0\, funksiya$ o`suvchi , funksiya o`zinig ekstremum qiymatiga xosila mavjud bo`lmagan va 0 ga teng nuqtalarda erishadi. $f(\dot{z}) = 0$ z=$\frac{1}{\sqrt{2}}$ f($\frac{1}{\sqrt{2}}$)= $\sqrt{2}$

$f(\dot{1})$ mavjud emas f(1)=1 minf(z)= f(1)=1

$$c_1 + c_2 - \int_0^t (\sqrt{1 - v_1{}^2} + \sqrt{1 - v_2{}^2})ds \leq c_1 + c_2 - \int_0^t 1ds = c_1 + c_2 - t = 0$$

t=$c_1 + c_2$ Demak uzog`i bilan T=$c_1 + c_2 + t_1$ vaqtda 1-xol uchun A qavariq to`plamda differensial o`yinni tugatish mumkin.

2-xol . $\Gamma_1$va Γ_2 O`rta perpendikularlar o`zaro kesishadi .Bu xol uchun xam differensial o`yinni tugatish mumkun.1)Xar ikkala quvlovchi $\pi$ strategiyasi bo`yicha xarakat qiladi. 2) $|u_i| \leq 1$ $|v| \leq 1$ O`yin ishtirokchilarining tezliklari 1 dan ortib ketmaydi 3)O`yinni tugatish mumkunligi: $e_i = \frac{y_0 - x_{i0}}{|y_{0-}x_{io}|}$, i=1,2

$\max(m_i - x_{i0}, e_i) = c_i$ bo`lsin $m_i \in M$

$$\sum_{i=1}^{2}(y(t)-x_i\,(t),e_i)=\sum_{i=1}^{2}(y(t)-x_{i0},e_i)$$

$$-\int_0^t\sum_{i=1}^{2}\,((v(s)-(v(s),e_i))e_i$$

$$-\,e_i\sqrt{1-|v(s)|^2+(v(s),e_i)^2}\,,e_i)ds$$

$$\leq c_1+c_2-\int_0^t\sum_{i=1}^{2}\sqrt{1-|v(s)|^2+(v(s),e_i)^2}\,ds$$

f(v)=$\sum_{i=1}^{2}\sqrt{1-|v|^2+(v,e_i)^2}$

1)f(v) uzluksiz 2)$|v|\leq 1$ yopiq va chegaralangan minf(v)=f(v_0)= α

ifodaning eng kichik qiymatini α Deb belgilaylik. $1-|v|^2\geq 0$, $(v,e_i)^2\geq 0$ =>, $\alpha\geq 0$. Endi $\alpha\neq 0$ ekanligini ko`rsatamiz .Faraz qilaylik $\exists v_0$ da $\alpha=0$ bo`lsin . => $1-|v_0|^2=0$, $(v_0,e_i)^2=0$,i=1,2 v_0 xar ikkala birlik vektorga perpendikular ekan,=> e_1 va e_2 vektorlar parallel .Biz 2-xolni e_1 va e_2 vektorlar o`zaro kesishsin deb boshlagan edik ,ular parallel bo`lib qoldi ziddiyatga keldik.Demak $\alpha=0$

deb qilgan farazimiz noto`g`ri => $\alpha>0$ ekan.

$$\leq c_1+c_2-\int_0^t\alpha ds=c_1+c_2-\alpha t=0$$

T=$\frac{c_1+c_2}{\alpha}$ d(0)= $\sum_1^2(y(0)-x_i\,(0),e_i)>0$

d(T)= $\sum_1^2(y(t)-x_i\,(t),e_i)\leq 0$ => $0<\tau\leq T$ vaqt oralig`ida $\exists\tau$ topiladiki $d_{i_0}(\tau)=0$ tenglik o`rinli bo`ladi. => y(τ) $-\,x_{i_0}(\tau)=e_{i_0}d_{i_0}(\tau)$ =0

y(τ) $=x_{i_0}$ 2-xol uchun xam o`yinni tugatish mumkun ekanligi ko`rsatildi.Xar ikkala xolat uchun xam berilgan differensial o`yinni tugatish mumkunligini ko`rsatdik, demak A qavariq to`plamda o`yinni tugatish mumkun.

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Ikkita quvlovchi va bir qochuvchili differensial o`yinlar

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