Tartiblangan fazolar $\ell^p(S)$

Tojinorova Maxliyo Murodjon qizi

TTYeSIda assistent

Axmedov Quanishbek Nizomiddinovich

TTYeSIda assistant

Mamajonova Malika Abdullo qizi

TTYeSIda talaba

Annotatsiya. *Ushbu maqolada umumiy topologiya va funksional analiz o'rtasidagi duch kelinadigan normallangan fazolarning yana bir muhim sinfi ko'rib chiqilgan. Ular o'lchovsiz Lebesg fazolari* $\ell^p(X, A, \mu)$ *bilan birinchi uchrashishni taminlaydi va keyingilari uchun zarur bo'lgan nazariy bog'lanishni uyg'unlashtiradi.*

Funktsional analiz va matematikaning tegishli sohalarida tartiblangan fazo vektor fazosi bo'lib, uning elementlari haqiqiy yoki kompleks sonlarning cheksiz ketma-ketliklaridan iborat. Ekvivalent ravishda bu funksiya fazosi bo'lib, uning elementlari natural sonlardan haqiqiy yoki kompleks sonlarning maydonigacha bo'lgan funksiyalardir. [1]

Kalit so'zlar: *Lebesg fazosi, Hyulder va Minkovskiy tengsizligi*

Asoslar. $1 \leq p \leq \infty$; *Hyulder*[1] *va Minkovskiy*[2] *tengsizligi*

Ta'rif.(ℓ^p *– Fazolar) Agar* $\mathbb{F} \in \{\mathbb{R}, C\}$, $0 < p < \infty$, $\mathbb{S}$ *to'plam va* $f : \mathbb{S} \to \mathbb{F}$ *bo'lsa, ko'rinib turibdiki,*

$$\|f\|_\infty = \sup_{s\in S} |f(s)| \in [0, \infty], \quad \|f\|_p = \left(\sum_{s\in S} |f(s)|^p\right)^{1/p} \in [0, \infty],$$

Bu yerda $\infty^{1/p} = \infty$ *va endi barcha p lar uchun* $p \in (0, \infty]$ *qo'yilsa,*

$$\ell^p(\mathbb{S}, \mathbb{F}) := \{\boldsymbol{f} \ : \ \mathbb{S} \to \mathbb{F}, \|f\|_p < \infty\}$$

---

[1] Otto Xolder (1859-1937), nemis matematigi. Tahlil va algebraga muhim hissa

[2] Herman Minkovski (1864-1909), nemis matematiki. Raqamlar nazariyasi, nisbiylik va boshqalarga qo'shgan hissasi

Lemma F.1.2 Barcha $p \in (0, \infty]$ va $f : \mathbb{S} \to \mathbb{F}$ lar uchun o'rinli:

(i) $\|f\|_p = 0$ agar va faqat agar $f = 0$ bo'lsa.

(ii) Barcha $c \in F$ lar uchun $\|cf\|_p = |c|\|f\|_p$ $(0 \cdot \infty = 0$ tushunchasi bilan)

(iii) Agar S to'plam chekli bo'lsa, $\ell^p(\mathbb{S}, \mathbb{F}) = \{f : \mathbb{S} \to \mathbb{F}\} = \mathbb{F}^{\mathbb{S}}$. Agar $\#S = 1$ bo'lsa, barcha $\|\cdot\|_p$ lar mos keladi.

Isbot.Oddiy.

Lemma F.1.3

(i) $(\ell^p(\mathbb{S}, \mathbb{F}), \|\cdot\|_p)$ lar uchun $p = 1$ va $p = \infty$ normallangan vektor fazolardir.

(ii) Agar $f \in \ell^p(\mathbb{S}, \mathbb{F})$ va $g \in \ell^\infty(\mathbb{S}, \mathbb{F})$ bo'lsa, unda

$$\left|\sum_{s \in S} f(s)g(s)\right| \le \|fg\|_1 \le \|f\|_1 \|g\|_\infty$$

Isbot. $\ell^p(\mathbb{S}, \mathbb{F})$ shubxasiz, skalyar ko'paytma ostida o'zgarmasdir va

$$\|f+g\|_\infty = \sup_s |f(s) + g(s)| \le \sup_s |f(s)| + \sup_s |g(s)| = \|f\|_\infty + \|g\|_\infty$$

$$\|f+g\|_1 = \sum_s |f(s) + g(s)| \le \sum_s (|f(s) + g(s)|) = \|f\|_1 + \|g\|_1$$

Shu tariqa $p \in \{1, \infty\}$ uchun $f, g \in \ell^p(\mathbb{S}, \mathbb{F})$ berilgan bo'lsa $f + g \in \ell^p(\mathbb{S}, \mathbb{F})$ bo'ladi, shunday qilib, $\ell^p(\mathbb{S}, \mathbb{F})$ bu $\mathbb{F}$- vektor fazodir va $\|\cdot\|_p$ undagi normani aniqlaydi. *(ii)* uchun bizga faqat $|f(s)g(s)| \le \|g\|_\infty |g(s)|$ kerak bo'ladi va bu yuqorida *(iii)* isbotlandi.

$1 < p < \infty$ uchun $\frac{1}{p} + \frac{1}{q} = 1$ orqali $q \in (1, \infty)$ligi yaqqol namoyon bo'ladi.

(bu ko'pincha foydali bo'lgan $pq = p + q$) ga teng). Qachonki, p, q birgalikda ko'rinsa, bu holatda ular ikki juftlik bo'lishi kerak. Biz buni $(1, \infty)$ va $(-\infty, 1)$ni ikki juftlik deb e'lon qilish orqali tabiiy yo'l bilan kengaytiramiz.

Izoh.1.4. $1 < p < \infty$ va q juftlik berilgan bo'lsin. U holda

(i) Barcha $f, g: \mathbb{S} \to \mathbb{F}$ lar uchun $\|fg\|_1 \leq \|f\|_p + \|g\|_q$ o'rinli (Hyulder tengsizligi)

(ii) Barcha $f, g: \mathbb{S} \to \mathbb{F}$ lar uchun $\|f + g\|_p \leq \|f\|_p + \|g\|_p$ o'rinli (Minkovskiy tengsizligi)

Isbot.*(i)* Aytaylik, $\|f\|_p, \|g\|_p$ chekli deb faraz qilaylik. $x \to e^x$ funksiya qavariq, $\frac{1}{p} + \frac{1}{q} = 1$ ekanligidan, $e^{a/p} e^{b/q} = exp\left(\frac{a}{p} + \frac{b}{q}\right) \leq \frac{e^a}{p} + \frac{e^b}{q}$; $\forall a, b \in \mathbb{R}$ deb qabul qilamiz.

$u, v > 0$ bo'lsa $e^a = u^p, e^b = v^q$ ekanligidan

$u \cdot v \leq \frac{u^p}{p} + \frac{v^q}{q}$ $\forall u, v \geq 0$ o'rinli.

(Ko'rinib turibdiki, $u = 0$ yoki $v = 0$ da tengsizlik bajarilishi)

(F1)da $u = |f(s)|$, $v = |g(s)|$ o'rniga qo'yilgan,

$|f(s)g(s)| \leq p^{-1}\|f\|_p^p + q^{-1}\|g\|_q^q$. $\|f\|_p = \|g\|_q = 1$

$f' = f/\|f\|_p \qquad g' = g/\|g\|_q$

$\|f'\|_p$=1=$\|g'\|_q$

$\|f'g'\|_1 \leq 1$

$\|f'g'\|_1 \leq \|f\|_p\|g\|_q$

$$\sum_s |f(s) + g(s)|^p \leq \sum_s (|f(s)| + |g(s)|)^p \leq \sum_s (2\max(|f(s)|, |g(s)|)^p)$$

$$\leq 2^p(\|f\|_p^p + \|g\|_q^q) < \infty$$

$\|f+g\|_p^p = \sum_s (|f(s)+g(s)|)^p \le \sum_s (|f(s)|+|g(s)|)|f(s)+$

$g(s)|^{p-1} \le (\|f\|_p + \|f\|_p)\||f+g|^{p-1}\|_q = \left(\|f\|_p + \|g\|_p\right)\|f+g\|_p^{p/q}$

F.1.5.Xulosa. Aytaylik, $1 < p < \infty$ bo'lsin. U holda

(i) $(\ell^p(\mathbb{S},\mathbb{F}), \|\cdot\|_p)$ bu normallangan vector fazodir.

(ii) Agar q pga ikkita $f \in \ell^p(\mathbb{S},\mathbb{F})$ va $g \in \ell^p(\mathbb{S},\mathbb{F})$ bo'lsa, u holda

$$\left|\sum_{s\in S} f(s)g(s)\right| \le \|fg\|_1 \le \|f\|_p\|g\|_q.$$

Foydalanilgan adabiyotlar:

1. Banach, Stefan; Mazur, S. (1933), "Zur Theorie der linearen Dimension", Studia Mathematica, **4**: 100–112.
2. Dunford, Nelson; Schwartz, Jacob T. (1958), Linear operators, volume I, Wiley-Interscience.
3. Topology for the working mathematician
Michael Muger
4. http://commons.wikimedia.org/wiki/File:Mug and Torus morph.gif